KSME International Journal, Vol. 12, No. 1, pp. 67~72, 1998

Fatigue Life Prediction by Statistical Approach Under Constant Amplitude Loading

Ouk Sub Lee*
(Received April 17, 1997)

In general, the experimental data of fatigue crack growth rates scatter very much even under identical experimental condition such as a constant amplitude loading condition. It is, thus, essential to take into account the data scatter of crack growth rates by using statistical approach for a reliable fatigue crack propagation analysis. In this study, fatigue crack propagation tests were conducted on a 1.02 mm-thick 2024-T3 aluminum alloy under a constant amplitude loading condition. The distribution of the fatigue crack propagation life is estimated by using the stochastic Markov chain model based on a modified Paris-Erdogan equation to consider the variability of the fatigue crack growth. The fatigue lives estimated by using the Markov chain model are found to be agreed well with the experimental results.

Key Words: Markov Chain, Fatigue Life, DC (Duty Cycle), CD (Cumulative Damage), A Modified Paris- Erdogan Equation, Fatigue Crack Growth Rate, Fatigue Crack Propagation Life, Aluminum Alloy, Data Scatter

1. Introduction

Since the large transportation mechanical structures such as ships and airplanes are generally operated under random variable loading conditions, the fatigue damage analysis is essential to provide the proper technical information in constructing the reliable and integrated ones. It is of noticeable that these structures consist mostly of very thin engineering materials comparing to other heavy structures such as the nuclear power pressure vessel. Therefore, it seems to be important to find out the precise fatigue crack growth behavior in very thin engineering materials. It is also necessary to generate the data-base from experiments on fatigue crack growth behavior of very thin specimens under varying constant amplitude loading conditions. One may utilize these data-bases for the analysis of structures under the random variable loading conditions. The variability of fatigue crack growth rate even under identical environmental condition needs a statistical model. Generally, the following three sources of variability in experimentally obtained fatigue crack growth data are commonly regarded as the most decisive (Sobczyk and Spencer, 1992): (1) the difference in material behavior among identically prepared specimens (due to difference in stress concentration at grain boundaries, effects of thermal processing, etc); (2) uncertainty in the fatigue and fracture process itself; (3) difference in environment among tests at the same load condition and with the same materials.

There are, in particular, two ways to consider and to estimate the variability of the fatigue crack growth. One is the estimation of crack growth life distribution from the Paris-Erdogan differential equation model in which we treated material constants, C, mas the random variables (Ishikawa and Tsurui, 1987). The other is the Markov chain model that is proposed by Bogdanoff-Kozin (1981, 1983), Kim and Kim (1995), and Kim and Shim (1996) as an example of the evolutionary probabilistic approach. In this study, we used the Markov chain model based on a modified Paris-Erdogan equation to combine the two technical methodologies together.

* Department of Mechanical Engineering, Inha University, Inchon 402-751, Korea

A modified Paris-Erdogan equation (Euw, Hertzberg and Roberts, 1972) used in this study is

$$\frac{da}{dN}=C(\Delta K_{eff})^{m} \tag{1}$$

where, $\frac{da}{dN}$: fatigue crack growth rate

C, m : random variables

$\Delta K_{eff}=\Delta\sigma\sqrt{\pi a\cdot\sec\left(\frac{\pi a}{W}\right)}(0.5+0.4R)$

R : stress ratio$\left(=\frac{\text{min. stress}}{\text{max. stress}}\right)$

Note that $0.5+0.4R$ (where $-0.1\leq R\leq 0.7$) is used to consider crack closure effect which commonly encountered in thin materials.

The principal purpose of this paper is, thus, to find an appropriate stochastic model and to evaluate reliability of this model for the fatigue crack growth analysis of a thin 2024-T3 aluminum alloy.

2. Background

Bogdonoff and Kozin (1985) used the Markov chain model so as to analyze statistically the fatigue cumulative damage process. They defined a duty cycle (DC) to be a repetitive period of operation in the life of a component during which damage can accumulate. They made the following assumptions ;

1. Damage states are discrete and labeled $j=1, 2, \cdots, b$ where, state b denotes replacement, or failure.
2. Increment in damage at the end of a DC depends in a probabilistic manner only on the amount of damage present at the start of the DC, on that DC itself, and is independent of how damage is accumulated up to the start of that DC.
3. Damage can only increase in a DC from the state occupied at the start of that DC to the state one unit higher.

If we define that P_j is the probability of remaining in state j during one step and q_j is the probability that in one step damage goes from state j to state $j+1$;

$p_j=\text{Prob}\{\text{remain in state } j \mid \text{initially in state } j\}$
$q_j=\text{Prob}\{\text{go to state } j+1 \mid \text{initially in state } j\}$.

The $(1\times b)$ row vector

$$p_0=\{\pi_1,\ \pi_2,\ \cdots,\ \pi_{b-1},\ 0\}$$

specifies the initial distribution of damage. Where, $\pi_j=\text{Prob}\{\text{damage is in state } j \text{ at time } x=0\}$, and $\sum_1^{b-1}\pi_j=1$, $\pi_b=0$. The assumption that, $\pi_b=0$, means that no component is in the failed state b initially. For the simple version of this model, the DC severity is defined by the following $(b\times b)$ probability transition matrix.

$$P=\begin{bmatrix} p_1 & q_1 & 0 & 0 & \cdots\cdots\cdots & 0 & 0 \\ 0 & p_2 & q_2 & 0 & \cdots\cdots\cdots & 0 & 0 \\ 0 & 0 & p_3 & q_3 & \cdots\cdots\cdots & 0 & 0 \\ 0 & 0 & p_4 & q_4 & \cdots\cdots\cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots\cdots\cdots & p_{b-1} & q_{b-1} \\ 0 & 0 & 0 & 0 & \cdots\cdots\cdots & 0 & 1 \end{bmatrix}$$

where, $p_j\geq 0$, $p_j+q_j=1$

In this matrix, note that all states are transient except for the last which is absorbing. The probability of being in state j at the time x is given by the $(1\times b)$ row vector.

$$p_x=\{p_x(1),\ p_x(2),\ p_x(3),\ \cdots,\ p_x(b)\}$$

where, $p_x(j)=\text{Prob}\{\text{damage is in state } j \text{ at time } x\}$

$$\sum_1^b p_x(j)=1,\quad p_x(j)\geq 0$$

We then have Markov property,

$$p_x=p_0P^x \tag{2}$$

For the fatigue cumulative damage, however, the Markov chain model of Bogdanoff and Kozin is based simply on the probabilistic process. Therefore, the model is not definite physical meaning of fatigue damage. Because of this reason, the crack growth law of the Paris-Erdogan is imported and its weakness can be made up for. In the Markov chain model, it is assumed that crack length δa increases by stage. Therefore, damage state j defines as

$$a_j=a_0+j\delta a,\quad j=0,\ 1,\ 2,\ \cdots,\ b \tag{3}$$

where, a_j is crack length in state j (mm), and a_0 is initial crack length (mm).

The probability of going to next state, q_j, can be defined as stress intensity factor function.

$$q_j = q(\Delta K_j) \tag{4}$$

In this approach, transient probability q_j (here, we assume q_j are independent of each loading step) can be obtained by using a modified Paris-Erdogan equation. The modified Paris-Erdogan equation is

$$\frac{da}{dN} = C(\Delta K_{eff})^m \tag{5}$$

It is assumed that m and C are random variables. However, they are considered to be contant for a single specimen but to show the dependency on each specimen. Eq. (5) may be rewritten in terms of advanced incremental crack length δ_a and mean value of duty cycle $E[\delta N]$

$$\frac{\delta a}{E[\delta N]} = C(\Delta K)_{eff})^m \tag{6}$$

where, E[variable] indicates mean value of variable.

Crack does not propagate during duty cycles $(\delta n - 1)$. But if δn-th cycle acts, then crack would propagate, and that probability is q. Therefore probability distribution of δN is

$$P[\delta N = \delta n] = f_{\delta N}(\delta n) = qp^{\delta n - 1} \tag{7}$$

Mean and variance of duty cycle number are the first order and the second order moments as follows, respectively :

$$E[\delta N] = \sum_{\delta n=0}^{\infty} \delta n f_{\delta N} = \sum_{\delta n=0}^{\infty} \delta n \, q(1-q)^{\delta n - 1} = \frac{1}{q} \tag{8}$$

$$\text{Var}[\delta N] = B(\delta N^2] - (E[\delta N])^2 = \frac{1-q}{q^2} \tag{9}$$

Using Eqs. (6), (7), and (8), the following transient probability q, which considers scatter of fatigue crack growth behavior is obtained as follows :

$$q = \frac{C}{\delta a}(\Delta K_{eff})^m \tag{10}$$

Table 1 Chemical composition of 2024-T3 Al alloy (wt %).

Si	Fe	Cu	Mn	Mg	Cr	Zn	Ti
0.11	0.23	4.46	0.58	1.44	0.04	0.03	0.02

Table 2 Mechanical properties of 2024-T3 Al alloy.

Yield strength (MPa)	Tensile strength (MPa)	Elongation (%)
324	442	16.7

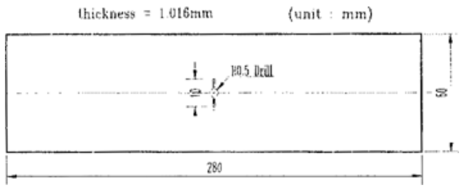

Fig. 1 Geometry of specimen.

3. Experiment

3.1 Specimen and experimental method

The material used was 2024-T3 Al alloy plates of 1. 02 mm thickness. Its chemical composition and mechanical property are shown in Table 1 and in Table 2, respectively.

The geometry of specimen is CCT (Center Cracked Tension) as shown in Fig. 1. The longitudinal direction of the specimen coincides with the rolling direction of the material. All fatigue crack growth tests were carried out under axial loading using a servo hydraulic testing machine of 10 ton capacity. The repeating frequency was 10 Hz. The stress range was $\Delta\sigma = 58.8$MPa, and the mean stress was $\sigma_m = 39.2$MPa. Hence the stress ratio was $R = 0.25$. The temperature of the specimen was room temperature. Crack growth was monitored using a traveling microscope, it can measure with accuracy of 0.01mm. The crack length was measured at the two tips of the crack on both sides of the specimen. The time interval of the measurement was 5000 cycles at the early stage of fatigue crack propagation period. The time intervals were reduced tremendously at the final stage of fatigue crack propagation to decrease measurement error of crack length.

Twenty five specimens were tested under identical experimental conditions.

4. Experimental Results and Discussion

4.1 Fatigue crack growth under constant amplitude loading

The crack lengths (a) are plotted against the number of cycles (N) in Fig. 2. Measurements were started at different initial crack lengths and the corresponding fatigue crack growth data were interpolated to adjust all curves with identical initial crack length of $a_0=7$ mm.

The relation between the crack growth rate, $\frac{da}{dN}$, and the effective stress intensity factor range, ΔK_{eff}, is shown in Fig. 3. $\frac{da}{dN}$ was evaluated as

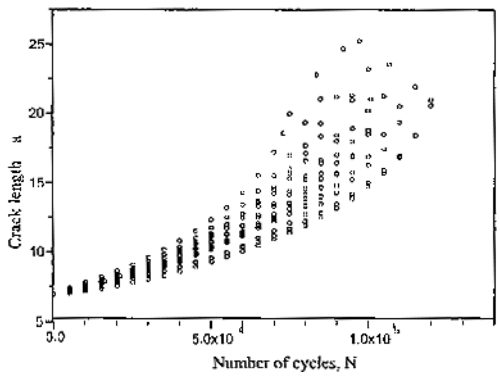

Fig. 2 Crack length plotted against the number of repeat cycles in Al 2024, $R=\frac{1}{4}$.

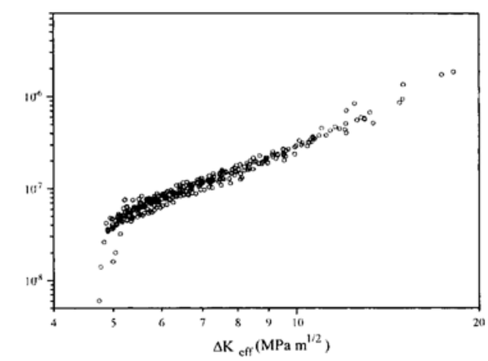

Fig. 3 Relationship between fatigue crack growth rate and stress intensity factor range, $R=\frac{1}{4}$.

$$\frac{da}{dN}=\frac{a_{i+1}-a_i}{N_{i+1}-N_i} \tag{11}$$

where, N_i and N_{i+1} are the number of cycles at which ith and $(i+1)$th measurements were made, respectively, and a_i and a_{i+1} are the values of crack lengths at $N=N_i$ and $N=N_{i+1}$, respectively. ΔK_{eff} was evaluated as

$$\Delta K_{eff}=U\Delta K_{app} = U\Delta\sigma\sqrt{\pi a\cdot\sec\left(\frac{\pi a}{W}\right)} \tag{12}$$

where, ΔK_{eff} is the effective stress intensity factor range,

ΔK_{app} is the applied stress intensity factor range,

U is the crack closure parameter, and

W is the specimen width.

Elber showed empirically for 2024-T3 Al alloy that

$$U=0.5+0.4R \tag{13}$$

where, R is the ratio of the mininum load to the maximum load. $-0.1\leq R\leq 0.7$

Substituting Eqs. (12) and (13) into Paris-Erdogan equation, it is possible to correlate crack growth rates with effective stress intensity factor range for different stress values as follows:

$$\frac{da}{aN}=C(\Delta K_{eff})^m = C\left[(0.5+0.4R)\Delta\sigma\sqrt{\pi a\cdot\sec\left(\frac{\pi a}{W}\right)}\right]^m \tag{14}$$

The modified Paris-Erdogan equation was applied to the data points of each specimen using the method of least squares. It is assumed that m

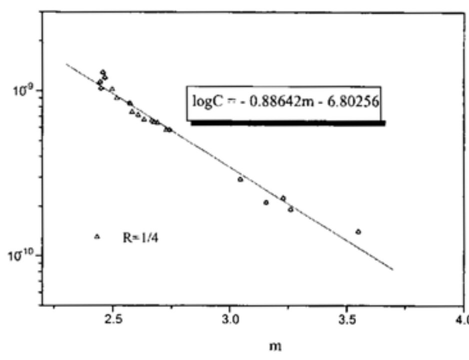

Fig. 4 Correlation between m and $\log C$ by using effective ΔK_{eff}, $R=\frac{1}{4}$.

and log C are random variables since the specimen-to-specimen variability of m and log C were clearly shown from experimental investigation. They both show approximately normal distributions.

Figure 4 shows the correlation between m and log C. It is seen that a strong negative correlation exists between m and log C. Therefore, if m (or log C) is generated as random variables by following normal distribution, then we could obtain values of m and log C to take account their strong negative correlation.

4.2 Fatigue life estimation

The two-parameter Weibull distribution such as Eq. (15) is known to fit fatigue crack propagation experimental data quite well (Sobczyk and Spencer, 1992, Bogdanoff and Kozin, 1985).

$$F(N)=1-\exp\left[-\left(\frac{N}{\theta}\right)^{\beta}\right] \tag{15}$$

where θ and β are functions of random variables m and C whose correlation is shown in Fig. 4. Markov chain model was constructed such that duty cycles are 1000 cycles and $\delta a=0.2$ mm. Figure 5 shows the edf (empirical distribute function) of the cycle number to reach $a=11$ mm, and the corresponding estimated result obtained from the proposed model which combines the Markov chain model and the modified Paris-Erdogan equation where C and m are random variables. Figure 6 shows the edf and estimated result at $a=17$ mm. The agreement between edf and estimated result seems to be excellent.

Figure 7 shows the comparison between edf's and estimated results of under $R=-\frac{1}{20}$ and $R=\frac{1}{2}$ loading conditions. The agreements between edf's and estimated results seem to be excellent even under different loading conditions such as $R=-\frac{1}{20}$ and $R=\frac{1}{2}$. The reliable fatigue life prediction may be obtained by using the proposed model in this study.

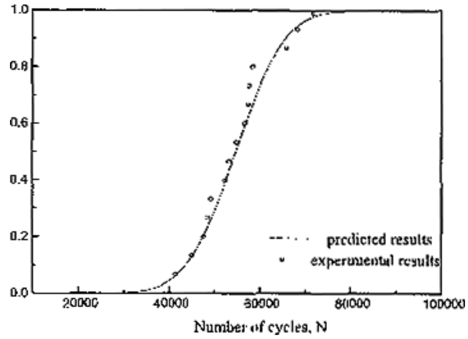

Fig. 5 Comparision between empirical results and the predicted fatigue life distribution using Markov Chain Model, $a=11$ mm, $R=\frac{1}{4}$.

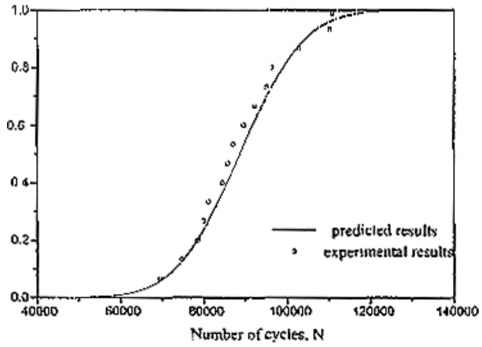

Fig. 6 Comparision between empirical results and the predicted fatigue life distribution using Markov Chain Model, $a=17$ mm, $R=\frac{1}{4}$.

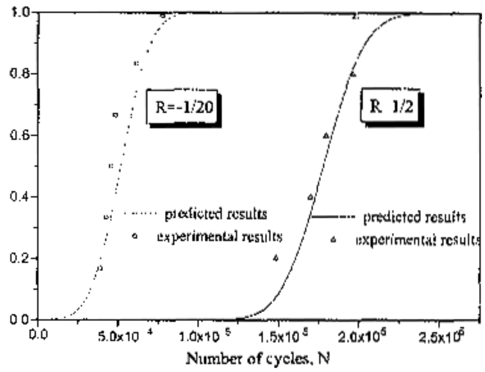

Fig. 7 comparison between edf's and the predicted fatigue life distribution using Markov Chain Model under different loading condition, $a=11$ mm.

5. Conclusion

In this study, a stochastic Markov-chain model is proposed to predict distribution of fatigue crack propagation lives of mechanical structural components under the constant amplitude loading conditions. The fatigue crack propagation tests were conducted and data scatter of fatigue crack propagation was considered by using a statistical model. Random variables m and C in a modified Paris-Erdogan model are imported to the statistical model. The transient probability, q, that considers (models) scatter of fatigue crack growth in terms of random variables C and m is modeled as

$$q = \frac{C}{\delta a}(\Delta K_{eff})^m$$

In this equation, note that ΔK_{eff} includes the effect of stress ratio. The distribution of fatigue crack propagation lives under the constant amplitude load conditions are estimated by using a stochastic Markov chain model based on a modified Paris-Erdogan equation. As a consequence, results of experiment and those estimated by using the modified Paris-Erdogan model are found to be agreed very well.

Acknowledgment

This research has been supported by both KOSEF through the Center for Transportation System of Yellow Sea at Inha University. The author appreciate the help of graduate students of Department of Statistics at Inha University, who calculated the results of Figs. 5, 6, and 7.

References

Bogdanoff J. L. and Kozin F., 1981, "A Critical Analysis of Some Probabilistic Models of Fatigue Crack Growth," *Engineering Fracture Mech.,* 14, 59∼89.

Bogdanoff J. L. and Kozin F., 1983, "On the Probabilistic Modeling of Fatigue Crack Growth," *Engineering Fracture Mech.,* 18, 623∼632.

Bogdanoff J. L. and Kozin F., 1985, "Probabilistic models of cumulative damage," In: *B-Models of Cumulative Damage,* pp. 63∼162. John Wiley & Sons, USA.

Ishikawa H. and Tsurui A., 1987, "Stochastic fatigue crack growth Model and Its Wide Applicability in Reliability-Based Design," *Current Japanese Material Research.,* 2, 45-58.

Kim J. K. and Kim D. S., 1995, *Trans. KSME* (A), Vol. 19. No. 8, pp. 1915∼1926.

Kim J. K. and Shim D. S., 1996, *Trans. KSME* (A), Vol. 20, No. 7, pp. 2159∼2166.

Sobczyk, K., and Spencer, B. F., 1992, "Random Fatigue from data to theory." In: *Scatter in Fatigue Data,* pp. 42∼43. Academic Press, San Diego.

Von Euw E. F. J., Hertzberg R. W. and Richard Roberts, 1972, "Delay effects in fatigue crack propagation," *ASTM STP* 513., 230∼259.